The internal dynamical variables would be γ^α and $S^{\alpha\beta}$, with

$$(\gamma^\alpha,\gamma^\beta)=4S^{\alpha\beta}, \tag{55}$$

$$(S^{\alpha\beta},\gamma^\delta)=g^{\alpha\delta}\gamma^\beta-g^{\beta\delta}\gamma^\alpha, \tag{56}$$

$$(S^{\alpha\beta},S^{\gamma\delta})=g^{\alpha\gamma}S^{\beta\delta}-g^{\beta\gamma}S^{\alpha\delta}-g^{\alpha\delta}S^{\beta\gamma}+g^{\beta\delta}S^{\alpha\gamma}. \tag{57}$$

The equations of motion would be

$$dx^\alpha/ds=\gamma^\alpha, \tag{58}$$

$$dP_\alpha/ds=eF_{\alpha\beta}\gamma^\beta, \tag{59}$$

$$d\gamma^\alpha/ds=4S^{\alpha\beta}P_\beta, \tag{60}$$

$$dS^{\alpha\beta}/ds=P^\alpha\gamma^\beta-P^\beta\gamma^\alpha. \tag{61}$$

We see from (58) that ds is an invariant parameter related to the proper time $d\theta$ by

$$(d\theta/ds)^2=\gamma^\alpha\gamma_\alpha. \tag{62}$$

Note that there are no constraints in this theory.

It is not difficult to solve these equations explicitly in the case of a free particle, and it is found that the Zitterbewegung is not of the same type as in the original Dirac particle. Namely, not only $\mathbf{q}$, but also t oscillates periodically as a function of s, so that $(d\mathbf{q}/dt)$ is not a sinusoidal function. It follows that this manifestly covariant system of equations is not a faithful model of the Dirac electron.

One may still ask whether the correct equations of motion (26) to (31) can be recast into a manifestly covariant form, with the proper time as an evolution parameter. In our opinion, this should not be possible, because Eqs. (26)–(31) have spurious solutions which are not Lorentz-invariant, and it is difficult to see how this could happen if they were equivalent to manifestly covariant equations.

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Unitary Symmetry and Electromagnetic Interactions*

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The general form of the electromagnetic interaction in the octet version of the proposed "higher symmetry" scheme based on the group SU_3 is derived. The result, which is applicable to an arbitrary multiplet, is expressed in an especially simple form by introducing the notion of U spin. Relations among electromagnetic form factors, mass splittings, decay amplitudes, and scattering amplitudes, previously obtained by various authors in the case of octets, are shown to follow immediately, as well as their generalizations to arbitrary multiplets. Where possible, comparisons are made with experiment.

I. INTRODUCTION

If the octet version[1] of the higher symmetry scheme based on the group SU_3 were exact, the known particles and resonances would form degenerate multiplets.[2] This degeneracy is not present in nature. However, it has been supposed that the deviations from the exact symmetry are due to some symmetry-breaking interactions which can be regarded as perturbations. Although no deep understanding of the symmetry-breaking interactions has been advanced, some results have been obtained which follow simply from the postulated transformation properties of the symmetry-breaking interactions. For example, Okubo[3] has obtained a "mass formula" by assuming the mass splittings transform like the hypercharge component of an octet. Similarly, the symmetry-breaking effects of the electromagnetic current have been considered by various authors[4] for the eight-dimensional multiplets.

In the present work we derive a concise expression for the most general form of the electromagnetic interaction in *any* representation of SU_3. The results derived previously for octets and their generalizations to arbitrary multiplets follow immediately from our formula. These results consist of relations between various electromagnetic form factors, mass splittings,

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[1] M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report, CTSL-20 (unpublished); Phys. Rev. **125**, 1067 (1962); and Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[2] For a current summary of the proposed multiplet assignments and further references, see L. Alvarez, M. Alston, M. Ferro-Luzzi, D. Huwe, G. Kalbfleisch, D. Miller, J. Murray, A. Rosenfeld, J. Shafer, F. Solmitz, and S. Woiciciki, Phys. Rev. Letters **10**, 184 (1963).
[3] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962); and Phys. Rev. Letters **4**, 14 (1963).
[4] S. Coleman and S. Glashow, Phys. Rev. Letters **6**, 423 (1961), N. Cabibbo and R. Gatto, Nuovo Cimento **21**, 872 (1961); and S. Okubo, Ref. 3.

decay amplitudes, and scattering amplitudes, which should be valid when other (nonelectromagnetic) deviations from the symmetry scheme are unimportant.

II. GENERAL FORM OF THE ELECTROMAGNETIC INTERACTIONS

Let us begin by considering the group U_3, whose nine generators, $F_\alpha{}^\beta$, have the following properties:

$$[F_\alpha{}^\beta,F_\rho{}^\sigma]=F_\alpha{}^\sigma\delta_\rho{}^\beta-F_\rho{}^\beta\delta_\alpha{}^\sigma\,, \tag{1}$$

$$F_\alpha{}^{\beta\dagger}=F_\beta{}^\alpha \quad (\alpha,\beta=1,2,3)\,. \tag{2}$$

From Eq. (1) it is clear that $F_1{}^1$, $F_2{}^2$, and $F_3{}^3$ can be simultaneously diagonalized. Their respective eigenvalues, f_1, f_2, and f_3, are integers forming the components of weight vectors, and the highest weight vector characterizes an irreducible representation of U_3. The identifications of these eigenvalues with particle quantum numbers are given by:

$$F_1{}^1=B+T_3+\tfrac{1}{2}Y\,, \tag{3}$$

$$F_2{}^2=B-T_3+\tfrac{1}{2}Y\,, \tag{4}$$

$$F_3{}^3=B-Y\,, \tag{5}$$

where B is the baryon number, Y is the hypercharge, and T_3 is the third component of isotopic spin. From these relations one finds the electric charge Q is given by

$$Q=F_1{}^1-\tfrac{1}{3}F_\alpha{}^\alpha. \tag{6}$$

Here, and in the following, it is to be understood that repeated indices are summed from 1 to 3.

From Eq. (6) it is clear that the electric charge, and hence the electromagnetic current, J_μ, have the transformation properties under U_3 of the component $T_1{}^1$ of a traceless tensor $T_\alpha{}^\beta$. Therefore, the most general expression for the electromagnetic current in a given representation of U_3 must be of the form[5]

$$J=j_1(F_1{}^1-\tfrac{1}{3}F_\alpha{}^\alpha)+j_2(F_\alpha{}^1F_1{}^\alpha+F_1{}^\alpha F_\alpha{}^1-\tfrac{2}{3}F_\alpha{}^\beta F_\beta{}^\alpha)\,, \tag{7}$$

where four-vector indices have been suppressed and j_1 and j_2 depend on the nonelectromagnetic interactions, which are assumed to be invariant under U_3.

Having this most general form, it only remains to explicitly compute the matrix elements for an arbitrary representation of U_3. For the first term in Eq. (7) the result is already given by Eq. (6). To facilitate the evaluation of the matrix elements of the second term let us notice that Eq. (1) implies

$$[F_\alpha{}^\alpha,F_\mu{}^\nu]=[F_\alpha{}^\beta F_\beta{}^\alpha,F_\mu{}^\nu]=0 \quad (\mu,\nu=1,2,3)\,. \tag{8}$$

Therefore, in any irreducible representation, $F_\alpha{}^\alpha$ and $F_\alpha{}^\beta F_\beta{}^\alpha$ must be represented by multiples of the identity. A direct calculation shows that in the representation of U_3 denoted by (f_1,f_2,f_3) we have

$$F_\alpha{}^\alpha=f_1+f_2+f_3\,, \tag{9}$$

$$F_\alpha{}^\beta F_\beta{}^\alpha=f_1{}^2+f_2{}^2+f_3{}^2+2(f_1-f_3)\,. \tag{10}$$

Using the above results, one finds directly that

$$\{F_1{}^\alpha,F_\alpha{}^1\}-\tfrac{2}{3}F_\alpha{}^\beta F_\beta{}^\alpha=\tfrac{1}{2}Q^2+\tfrac{4}{3}(f_1+f_2+f_3)Q-2U(U+1)$$
$$+\tfrac{1}{3}[f_1{}^2+f_2{}^2+f_3{}^2+2(f_1-f_3)]-\tfrac{1}{9}(f_1+f_2+f_3)^2 \tag{11}$$

where we have introduced the total "isocharge" spin U associated with an SU_2 subgroup of U_3.[6]

This "isocharge" spin U, which is more useful in the present context that the usual isotopic or "isohypercharge" spin, is associated with the SU_2 subgroup having generators

$$U_1=\tfrac{1}{2}(F_2{}^3+F_3{}^2)\,, \tag{12}$$

$$U_2=\frac{i}{2}(F_2{}^3-F_3{}^2)\,, \tag{13}$$

$$U_3=\tfrac{1}{2}(F_3{}^3-F_2{}^2)\,. \tag{14}$$

These operators clearly generate an SU_2 subgroup since their commutation relations, which follow from Eq. (1), are

$$[U_\alpha,U_\beta]=i\epsilon_{\alpha\beta\gamma}U_\gamma\,. \tag{15}$$

The simplicity achieved by introducing U spin, which becomes more apparent below, is due to the fact that the electromagnetic interaction commutes with U spin. Hence, all the members of a U-spin multiplet have the same electromagnetic charge. (In the more familiar case of isotopic spin they all have the same hypercharge.)

To further simplify the final result, note that the form of the matrix element of Eq. (7) in a given irreducible representation does not depend in an essential way on all of the three numbers, f_1, f_2, and f_3, but only on the differences $p=(f_1-f_2)$ and $q=(f_2-f_3)$, which designate the irreducible representations of the subgroup SU_3. Consequently, the most general expression for the electromagnetic current in the irreducible representation of SU_3 denoted by (p,q) must be of the form

$$J=j_1Q+j_2\{2U(U+1)-\tfrac{1}{2}Q^2-\tfrac{4}{3}(p-q)Q$$
$$-\tfrac{1}{3}p(p+2)-\tfrac{1}{3}q(q+2)+\tfrac{1}{9}(p-q)^2\}\,. \tag{16}$$

Next, let us generalize this result to the matrix elements of products of the electromagnetic current in an arbitrary representation (p,q). The matrix element of the current taken twice must have the transformation properties of the component $T_{11}{}^{11}$ of a tensor $T_{\alpha\mu}{}^{\beta\nu}$ since this is how the charge squared transforms. There are six such independent tensors which can be formed

[5] For explicit proofs, see S. Okubo, Ref. 3, and H. Goldberg and Y. Lehrer-Ilamed, J. Math. Phys. 4, 501 (1963).

[6] The usefulness of U spin has been emphasized by S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters **10**, 361 (1963). This reference contains a summary of U-spin assignments differing only trivially from those used in the present work.

from products of $\delta_\mu{}^\nu$, $F_\mu{}^\nu-\frac{1}{3}F_\alpha{}^\alpha\delta_\mu{}^\nu$, and $\{F_\mu{}^\alpha,F_\alpha{}^\nu\}-\frac{2}{3}F_\alpha{}^\beta F_\beta{}^\alpha\delta_\mu{}^\nu$. Therefore, the most general expression for the matrix element of the two-current product in the representation (p,q) is

$$\begin{aligned}J\times J&=j_1+j_2Q+j_3\{2U(U+1)-\tfrac{1}{2}Q^2-\tfrac{4}{3}(p-q)Q\\&-\tfrac{1}{3}p(p+2)-\tfrac{1}{3}q(q+2)+\tfrac{1}{9}(p-q)^2\}+j_4Q^2+j_5Q\\&\times\{2U(U+1)-\tfrac{1}{2}Q^2-\tfrac{4}{3}(p-q)Q-\tfrac{1}{3}p(p+2)\\&-\tfrac{1}{3}q(q+2)+\tfrac{1}{9}(p-q)^2\}+j_6\{2U(U+1)-\tfrac{1}{2}Q^2\\&-\tfrac{4}{3}(p-q)Q-\tfrac{1}{3}p(p+2)-\tfrac{1}{3}q(q+2)+\tfrac{1}{9}(p-q)^2\}^2,\end{aligned}\quad(17)$$

where j_1 to j_6 depend on the nonelectromagnetic interactions, which are assumed to be invariant under SU_3. By continuing in this line of reasoning, one obtains the general form of the matrix element of the product of arbitrarily many currents in any irreducible representation of SU_3.

III. CONCLUSION

To illustrate how simply one can obtain relations among form factors, mass splittings, decay amplitudes, and scattering amplitudes, using the general form of the electromagnetic interaction derived above, let us rederive some results previously obtained by several authors[4] for octets and give their generalizations.

Since the eight baryons N(939 MeV), Λ(1115 MeV), Σ(1193 MeV), and Ξ(1315 MeV) are assumed to belong to the (1,1) representation of SU_3, their nine electromagnetic currents are expressed through Eq. (16) in terms of just two independent currents. Consequently, there are seven relations among the baryon currents. To obtain these relations we note that the baryons form U-spin multiplets as follows: Σ^+ and p form a doublet, Ξ^0, $(\Sigma^0-\sqrt{3}\Lambda)/2$, and n form a triplet, $(\Lambda+\sqrt{3}\Sigma^0)/2$ forms a singlet, and Ξ^- and Σ^- form a doublet. Now simply using Eq. (16), one finds the following relations among the baryon currents:

$$\begin{aligned}J(p)+J(\Xi^-)&=J(\Sigma^+)+J(\Sigma^-)=2J(\Sigma^0)=-2J(\Lambda)\\&=2/\sqrt{3}J(\Lambda\Sigma^0)=-J(n)=-J(\Xi^0),\end{aligned}\quad(18a)$$

$$J(p)+J(\Sigma^-)=J(\Sigma^+)+J(\Xi^-).\quad(18b)$$

As a special case, these relations apply to the static values of the form factors, namely, charges and magnetic moments. Clearly, charges satisfy the relations trivially, but the lack of data do not permit a check of the relations between magnetic moments. The only data bearing on these relations are the recent measurements of μ_Λ. The experimental values reported are[7] $\mu_\Lambda=-1.5\pm0.5$ and[8] $\mu_\Lambda=0.0\pm0.6$ nucleon magnetons, which are to be compared with the prediction of Eqs. (18), that $\mu_\Lambda=\frac{1}{2}\mu_n\simeq-0.95$ nucleon magnetons.

In the case of bosons, charge conjugation requires $j_2=0$ in Eq. (16). Hence, the boson currents are just proportional to the charge. For the pseudoscalar meson octet comprised of K(496 MeV), η(548 MeV), and π(140 MeV), the vector meson octet comprised of K^*(888 MeV), ω(785 MeV), and ρ(750 MeV), and the vector meson singlet φ(1020 MeV),[9] Eq. (16) implies the only nonvanishing currents are related by

$$J(K^+)=J(\pi^+)=-J(K^-)=-J(\pi^-),\quad(19)$$

$$J(K^{*+})=J(\rho^+)=-J(K^{*-})=-J(\rho^-).\quad(20)$$

The meson-baryon resonances $N_{3/2}^*$(1238 MeV), Y_1^*(1385 MeV), $\Xi_{1/2}^*$(1530 MeV), and the conjectured Ω^-($\sim$1676 MeV), have been tentatively identified with the (3,0) representation of SU_3.[10] In this representation the U-spin and charge are linearly related by $U=1-\frac{1}{2}Q$ so that Eq. (16) implies these meson-baryon currents are simply proportional to the charge. Consequently, there are nine relations among the ten currents. These relations imply that resonances with equal charges have equal form factors and that these form factors are equally spaced along the charge axis. In addition, the form factors vanish identically for the neutral resonances.

The meson-baryon resonances $N_{1/2}^*$(1512 MeV), Y_0^*(1520 MeV), Y_1^*(1660 MeV), and a conjectured $\Xi_{1/2}^*$($\sim$1600 MeV) are supposed to belong to the (1,1) representation. Consequently, their currents satisfy the same relations as the corresponding baryon currents, i.e., Eqs. (18).

The very short lifetimes of these resonances preclude checking any of the above relations for static moments. However, when electromagnetic particle-antiparticle pair production processes become experimentally feasible, perhaps the relations between form factors can be investigated.

Relations among electromagnetic mass splittings follow directly from using Eq. (17) in the computation of self-energy diagrams. For the baryon octet one finds the following relations:

$$\begin{aligned}\delta M(\Xi^-)-\delta M(\Xi^0)&=\delta M(\Sigma^-)-\delta M(\Sigma^+)\\&+\delta M(p)-\delta M(n),\end{aligned}\quad(21a)$$

$$\begin{aligned}2\sqrt{3}\delta M(\Sigma^0\Lambda)&=3\delta M(\Lambda)+\delta M(\Sigma^0)\\&-2\delta M(n)-2\delta M(\Xi^0),\end{aligned}\quad(21b)$$

where $\delta M(\Sigma^0\Lambda)$ is the $\Sigma^0\leftrightarrow\Lambda$ transition mass. Other relations also follow, but they are not useful since the large mass splittings due to strong interactions do not cancel out as they do in Eqs. (21). [These effects

[7] R. L. Cool, E. W. Jenkins, T. F. Kyeia, D. A. Hill, L. Marshall, and R. A. Schluter, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 345.

[8] W. Kerman, T. B. Novey, S. D. Warshaw, and A. Wattenberg, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 347.

[9] It has been suggested that mixtures of the observed ω and φ mesons should be identified with the singlet and octet. [See, for example, J. J. Sakurai, Phys. Rev. **132**, 434 (1963)]. This refinement of the octet symmetry model is immaterial to the present considerations.

[10] For a discussion of some problems associated with this identification see R. J. Oakes and C. N. Yang, Phys. Rev. Letters **11**, 174 (1963).

cancel out of Eq. (21b) only to the extent that the first order mass formula[3] is satisfied.] Experimentally[11] the right and left sides of Eq. (21a) are (6.7±0.5) MeV and (7.4±9.2) MeV, respectively.

If one considers the contributions from self-energy graphs in which only the baryons themselves appear as intermediate states, one finds more restrictive relations. Considering only such graphs leads to essentially Eq. (17) with $j_1=j_2=j_3=0$, in which case we have the additional relations

$$\delta M(n)-\delta M(p)=\delta M(\Sigma^0)-\delta M(\Sigma^+)\,, \quad (22a)$$

$$\delta M(\Sigma^-)-\delta M(\Sigma^0)=\delta M(\Xi^-)-\delta M(\Xi^0)\,, \quad (22b)$$

$$3\delta M(\Lambda)+\delta M(\Sigma^0)=2\delta M(n)+2\delta M(\Xi^0)\,. \quad (22c)$$

Comparing these with experimental masses[11] shows the hypothesis is not justified.

For the pseudoscalar and vector meson octets relations (21) and (22) hold for corresponding states under the corresponding assumptions. Equation (21a) is trivially satisfied due to charge conjugation invariance. For the pseudoscalar mesons Eq. (22) is clearly not satisfied.

The electromagnetic mass splittings for the meson-baryon resonances belonging to the (3,0) representation also follow from Eq. (17). In this case one finds the relation $U=1-\frac{1}{2}Q$ simplifies Eq. (17) greatly. It reduces to a quadratic polynomial in the charge, which implies

$$\delta M(N_{3/2}{}^{*++})+3\delta M(N_{3/2}{}^{*0})=3\delta M(N_{3/2}{}^{*+})+\delta M(N_{3/2}{}^{*-})\,, \quad (23a)$$

$$\delta M(N_{3/2}{}^{*+})-\delta M(N_{3/2}{}^{*0})=\delta M(Y_1{}^{*+})-\delta M(Y_1{}^{*0})\,, \quad (23b)$$

$$\delta M(N_{3/2}{}^{*0})-\delta M(N_{3/2}{}^{*-})=\delta M(Y_1{}^{*0})-\delta M(Y_1{}^{*-})=\delta M(\Xi_{1/2}{}^{*0})-\delta M(\Xi_{1/2}{}^{*-})\,. \quad (23c)$$

For the meson-baryon resonances belonging to the (1,1) representation, relations (21) hold for corresponding states. Unfortunately, such sum rules for the resonances are only of academic interest because of their short lifetimes.

Amplitudes for decays of the type vector meson → pseudoscalar meson+photon can be related by noting that the pseudoscalar mesons form U-spin multiplets as follows: π^+ and K^+ form a doublet, $\bar{K}^0$, $(\pi^0-\sqrt{3}\eta)/2$, and K^0 form a triplet, $(\eta+\sqrt{3}\pi^0)/2$ forms a singlet, and K^- and π^- form a doublet. The vector mesons form U-spin multiplets in the obvious corresponding manner.

[11] W. H. Barkas and A. H. Rosenfeld, University of California Radiation Laboratory Report, UCRL–8030 Rev. (April 1963 Edition) (unpublished).

Then from Eq. (16) one finds

$$\langle\gamma\pi^+|\rho^+\rangle=\langle\gamma\pi^0|\rho^0\rangle=\frac{1}{\sqrt{3}}\langle\gamma\eta|\rho^0\rangle=\frac{1}{\sqrt{3}}\langle\gamma\pi^0|\omega\rangle=-\langle\gamma\eta|\omega\rangle=\langle\gamma K^+|K^{*+}\rangle=-\tfrac{1}{2}\langle\gamma K^0|K^{*0}\rangle\,. \quad (24)$$

Here we have also assumed invariance under charge conjugation.

Several relations among Compton scattering amplitudes also follow directly from Eq. (17). For example, in the case of the baryons we have the following relations:

$$\langle\gamma p|\gamma p\rangle=\langle\gamma\Sigma^+|\gamma\Sigma^+\rangle\,, \quad (25a)$$

$$\langle\gamma n|\gamma n\rangle=\langle\gamma\Xi^0|\gamma\Xi^0\rangle\,, \quad (25b)$$

$$\langle\gamma\Sigma^-|\gamma\Sigma^-\rangle=\langle\gamma\Xi^-|\gamma\Xi^-\rangle\,, \quad (25c)$$

$$2\langle\gamma\Lambda|\gamma\Sigma^0\rangle=\sqrt{3}\langle\gamma\Sigma^0|\gamma\Sigma^0\rangle-\sqrt{3}\langle\gamma\Lambda|\gamma\Lambda\rangle\,, \quad (25d)$$

$$\sqrt{3}\langle\gamma\Lambda|\gamma\Sigma^0\rangle=\langle\gamma\Sigma^0|\gamma\Sigma^0\rangle-\langle\gamma n|\gamma n\rangle\,, \quad (25e)$$

$$\langle\gamma\Lambda|\gamma\Sigma^0\rangle=\sqrt{3}\langle\gamma\Lambda|\gamma\Lambda\rangle-\sqrt{3}\langle\gamma n|\gamma n\rangle\,. \quad (25f)$$

Clearly, using the general form of the electromagnetic interaction given above, the analysis can be continued to obtain relations among many other amplitudes involving photons.

To summarize, the study of electromagnetic interactions within the framework of the octet symmetry scheme can be considerably simplified by introducing the concept of U spin, which allows one to express electromagnetic interactions in the compact form derived above. However, the conclusions which follow are difficult to test experimentally at present and consequently offer little hope of providing a test of the validity of the octet symmetry model. *Note added in proof*. Recent data on the mass difference $\delta M(\Xi^-)-\delta M(\Xi^0)$, which reduce the uncertainty to about 2 MeV, are in agreement with Eq. (21a). [Proceedings of the Siena International Conference on Elementary Particles (to be published)].

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